# Preface

Welcome to Workshop on Variational Analysis and Nonsmooth Optimization [in conjunction of celebrating Terry Rockafellar's 90th birthday], organized by the Department of Applied Mathematics of The Hong Kong Polytechnic University.

This workshop is to foster the sustained development in nonsmooth optimization and variational analysis. 18 invited speakers including world leading researchers and active young researchers will attend the workshop from USA, Europe, Singapore, Mainland China. It will encourage international collaboration and provide an opportunity for young researchers to learn the current state of the art in optimization and variational analysis. We would like to take this opportunity to thank all the speakers and participants for their support to the conference.

We are grateful to the members of the organizing committee for their hard work. We are also grateful to the staff members of the Department of Applied Mathematics for their time and effort in making this workshop possible.

The workshop gratefully acknowledges the patronage of The Hong Kong Polytechnic University, and the financial support of CAS AMSS-PolyU Joint Laboratory of Applied Mathematics (JLab) and Research Center for Intelligent Operations Research.

Organizing Committee

# Acknowledgments

We are very grateful for the support and contributions to the workshop given by

The Hong Kong Polytechnic University (PolyU)

Faculty of Science, PolyU

Department of Applied Mathematics, PolyU

CAS AMSS-PolyU Joint Laboratory of Applied Mathematics (JLab)

Research Center for Intelligent Operations Research

We would also like to thank the staff members in the Department of Applied Mathematics who have worked together in handling the various arrangements related to the conference. Their assistance has been indispensable.

# Organizing Committees

### Co-chairs

- Xiaojun Chen (PolyU)
- Defeng Sun (PolyU)

### Members

- Xiaoxia Liu (PolyU)
- Ting-kei Pong (PolyU)
- Houduo Qi (PolyU)
- Xiaoqi Yang (PolyU)
- Yancheng Yuan (PolyU)
- Zaikun Zhang (PolyU)

# Conference Highlights

## Conference Venue (CHANGED)

**Room HJ202, The Hong Kong Polytechnic University.**

Please refer to the campus map. Please note that smoking is not allowed on the PolyU campus.

## Opening Ceremony

The opening ceremony will be held in **Room HJ202** at **9:00** in the morning of **02 December 2024**.

## Registration & Reception & Information Desk

Registration desk will be set up outside **Room HJ202** from **17:30-19:00 on 01 December 2024**. A notice board will be available nearby the registration desk. Updated information and announcement about the conference will be posted on the notice board.

## On-campus Internet Access

Desktop computers are available for internet access in room M302. The helper can take you to M302.

Wi-Fi access is provided during the workshop period. Please use the login name and password listed below.

Network: PolyUWLAN<br>
Username: gstama<br>
Password: Polyu4ever

## Registered participants are entitled to:

- Attendance to all conference sessions
- Invitation to the welcome reception
- Invitation to the opening ceremony
- Invitation to the lunch on 04 December and banquet on 03 December
- Coffee breaks
- A free copy of the Conference Abstracts and Conference Programme

## Lunch

Lunch on 04 December will be provided to all registered conference participants. Lunch tickets are attached to your official conference name tag. Be sure to bring along your tickets when you attend the lunch. In addition, there are a number of cafeterias available on campus. Please refer to the on-campus cafeteria map.

The nearby Tsim Sha Tsui East area, which is within walking distance from PolyU, provides a variety of teahouse, restaurants and fast food services. Please refer to the map of TST East.

## Banquet

The Conference Banquet will be held on 03 December 2024 at 18:00. A banquet ticket is attached to your official conference name tag. Be sure to bring along your banquet ticket when you attend the banquet.

## Group Photo

Conference Group Photo will be taken on 02 December 2024 after the Opening Ceremony. Please refer to the campus map.

**Schedule of Workshop (brief)**

| **01 Dec (SUN)** | | **02 Dec (MON)** | **03 Dec (TUE)** | **04 Dec (WED)** |
|---|---|---|---|---|
| | **9:00-9:10** | **Opening** | | |
| | **9:10-9:50** | Invited Talk | Invited Talk | Invited Talk |
| | **9:50-10:30** | Invited Talk | Invited Talk | Invited Talk |
| | **10:30-11:00** | Coffee Break | Coffee Break | Coffee Break |
| | **11:00-11:40** | Invited Talk | Invited Talk | Invited Talk |
| | **11:40-12:20** | Invited Talk | Invited Talk | Invited Talk |
| | | Lunch | Lunch | Lunch |
| | **14:20-15:00** | Invited Talk | Invited Talk | Invited Talk (14:00 – 14:40) |
| | **15:00-15:40** | Invited Talk | Invited Talk | Invited Talk (14:40 – 15:20) |
| | **15:40-16:00** | Coffee Break | Coffee Break | **Closing** (15:20 – 15:30) |
| | **16:00-17:00** | PolyU PhD students Poster session | PolyU PhD students Poster session | |
| Registration & Reception (17:30-19:00) | | | | |
| | **18:00-20:00** | | **Banquet** | |

## Schedule of Workshop

| | 02 December (Monday) | December 03 (Tuesday) | 04 December (Wednesday) |
|---|---|---|---|
| **9:00-9:10** | **Opening Ceremony** [HJ202]<br>Group Photo | | |
| | **Invited talk** [HJ202]<br>**Chair:** Defeng Sun | **Invited talk** [HJ202]<br>**Chair:** Xiaojun Chen | **Invited talk** [HJ202]<br>**Chair:** Xiaoqi Yang |
| **9:10-9:50** | Terry Rockafellar | Jong-Shi Pang | Yurii Nesterov |
| **9:50-10:30** | Boris Mordukhovich | Kim-Chuan Toh | Hélène Frankowska |
| **10:30-11:00** | Coffee Break | Coffee Break | Coffee Break |
| | **Invited talk** [HJ202]<br>**Chair:** Houduo Qi | **Invited talk** [HJ202]<br>**Chair:** Ting-kei Pong | **Invited talk** [HJ202]<br>**Chair:** Zaikun Zhang |
| **11:00-11:40** | Radu Ioan Boţ | Marc Teboulle | Jie Sun |
| **11:40-12:20** | Samir Adly | Andrew Eberhard | Johannes O. Royset |
| | Lunch | Lunch | Lunch |
| | **Invited talk** [HJ202]<br>**Chair:** Xiaoxia Liu | **Invited talk** [HJ202]<br>**Chair:** Yancheng Yuan | **Invited talk** [HJ202]<br>**Chair:** Defeng Sun |
| **14:20-15:00** | Claudia Sagastizábal | Mikhail Solodov | Michael Hintermüller<br>(14:00 – 14:40) |
| **15:00-15:40** | Zhiping Chen | Chao Ding | Ying Cui<br>(14:40 – 15:20) |
| **15:40-16:00** | Coffee Break | Coffee Break | **Closing Ceremony**<br>(15:20 – 15:30) |
| **16:00-17:00** | PolyU PhD students<br>Poster session | PolyU PhD students<br>Poster session | |
| | | | |
| **18:00-20:00** | | **Banquet** | |

# Invited Talks

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## 02 December 2024, Monday

**Chair:** Defeng Sun **[HJ202]**

09:10-09:50 **Terry Rockafellar** (University of Washington)
*The Surprising Importance of Convexity in Nonconvex Variational Analysis*

09:50-10:30 **Boris Mordukhovich** (Wayne State University)
*Variational Convexity in Variational Analysis and Optimization*

**Chair:** Houduo Qi **[HJ202]**

11:00-11:40 **Radu Ioan Boţ** (Faculty of Mathematics, University of Vienna)
*Fast Convergent Algorithms for Convex Optimization Problems with Linear Constraints*

11:40-12:20 **Samir Adly** (Universitéde Limoges)
*The Limit Dynamic of the Su-Boyd-Candès Inertial Gradient System: A High-Resolution Analysis of Accelerated Optimization*

**Chair:** Xiaoxia Liu **[HJ202]**

14:20-15:00 **Claudia Sagastizábal** (IMECC-University of Campinas)
*A Bundle-like View of Splitting Methods*

15:00-15:40 **Zhiping Chen** (Xi'an Jiaotong University)
*A Unifying Framework for Markov Decision Process Models with a Bayesian Composite Risk Approach*

15:40-16:00 **Coffee Break**

16:00-17:00 **PolyU PhD students Poster session**

# Invited Talks

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## 03 December 2024, Tuesday

**Chair:** Xiaojun Chen **[HJ202]**

09:10-09:50 **Jong-Shi Pang** (University of Southern California)
*Heaviside Composite Optimization: A new paradigm in optimization*

09:50-10:30 **Kim-Chuan Toh** (National University of Singapore)
*A Low-rank ALM for Doubly Nonnegative Relaxations of Mixed-binary QP*

**Chair:** Ting-kei Pong **[HJ202]**

11:00-11:40 **Marc Teboulle** (Tel Aviv University)
*Proximal Gradient Methods in Nonsmooth Nonconvex Minimax: A Unified Convergence Analysis Framework*

11:40-12:20 **Andrew Eberhard** (Royal Melbourne Institute of Technology)
*On the Theory of Maximal Monotone Operators in General Banach Spaces*

**Chair:** Yancheng Yuan **[HJ202]**

14:20-15:00 **Mikhail Solodov** (Institute for Pure and Applied Mathematics (IMPA))
*Descent Sequences in Weakly Convex Optimization*

15:00-15:40 **Chao Ding** (Chinese Academy of Sciences)
*A New Dual Semismooth Newton Method for Polyhedral Projections*

15:40-16:00 **Coffee Break**

16:00-17:00 **PolyU PhD students Poster session**

18:00-20:00 **Banquet**

# Invited Talks

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## 04 December, Wednesday

**Chair:** Xiaoqi Yang **[HJ202]**

09:10-09:50 **Yurii Nesterov** (Corvinus University and partly SDS CUHK (SZ), Professor emeritus at UCLouvain)
*New IPM for Linear Optimization Based on Affine-Scaling Directions in Parabolic Target Space*

09:50-10:30 **Hélène Frankowska** (CNRS, Institut de Mathématiques de Jussieu - Paris Rive Gauche, and Sorbonne Université)
*Control Systems on Wasserstein Spaces*

**Chair:** Zaikun Zhang **[HJ202]**

11:00-11:40 **Jie Sun** (Curtin University)
*The Elicited Progressive Decoupling Algorithm: on the Rate of Convergence and the Choice of Parameters*

11:40-12:20 **Johannes O. Royset** (University of Southern California)
*Approximations of Rockafellians, Lagrangians, and Dual Functions*

**Chair:** Defeng Sun **[HJ202]**

14:00-14:40 **Michael Hintermüller** (Humboldt-Universität zu Berlin)
*A Neural Network Approach to Learning Solutions of a Class of Elliptic Variational Inequalities*

14:40-15:20 **Ying Cui** (University of California, Berkeley)
*Optimization with Superquantile Constraints: A Fast Computational Approach*

15:20-15:30 **Closing Ceremony**

# The Surprising Importance of Convexity in Nonconvex Variational Analysis

Terry Rockafellar

University of Washington

**Abstract.** Many years of effort have gone into extending convex analysis to the much larger territory of variational analysis. Tangent cones and normal cones had to be defined more generally to work for nonconvex sets and then be applied to epigraphs of functions and the graphs of set-valued mappings. That resulted in a satisfyingly complete first-order theory, but second-order variational analysis has continued to present mysteries despite important progress on many fronts. Researchers have understood that second-order theory of nonconvex functions can't be expected to work effectively without making some suitable assumptions on the functions to be considered, and it has been hard to see where exactly the natural boundary for that lies. But now that boundary has come clearly into view as the presence or absence of the property of prox-regularity.

Prox-regularity involves a real parameter that can admitted at all levels, positive, negative and zero. When articulated that way, it corresponds at those levels with reversed sign to uniform quadratic growth properties which are important in themselves. Those properties are tied in turn to a variational convexity spectrum through which local behavior of subgradients and function values can be reduced to a picture in just convex analysis.

Amazingly, any point-subgradient pair for any lsc proper function has associated with it a "convexity modulus" value which determines whether prox-regularity is present, and if so, gives the exact and same threshold for the available level of several properties crucial to second-order variational analysis. Furthermore, tests based on new derivative concepts are able now to assess that modulus value.

# Variational Convexity in Variational Analysis and Optimization

Boris Mordukhovich

Wayne State University

**Abstract.** While the notions of variational and strong variational convexity have been introduced by Terry Rockafellar quite recently, the importance of them has been already recognized and documented in finite-dimensional variational analysis and optimization. This talk presents new results in this direction in both finite-dimensional and infinite-dimensional spaces. We provide comprehensive characterizations of these fundamental notions by developing novel techniques, which are essentially different from the known ones in finite dimensions. As a consequence of the obtained characterizations, new quantitative and qualitative relationships between strong variational convexity and tilt stability of local minimizers are established in appropriate frameworks of Banach spaces. The talk is based on joint work with P. D. Khanh, V. V. H. Khoa and V. T. Phat.

# Fast Convergent Algorithms for Convex Optimization Problems with Linear Constraints

Radu Ioan Boţ

Faculty of Mathematics, University of Vienna

**Abstract.** In this talk we discuss in the setting of real Hilbert spaces fast convergent algorithms for the minimization of a convex function under linear equality/inequality constraints.

In the case of a smooth objective function, we consider a second-order dynamical system with asymptotically vanishing damping term, formulated in terms of the Augmented Lagrangian associated with the minimization problem. The time discretization leads to an inertial algorithm with a general rule for the inertial parameters covering the classical ones of Nesterov, Chambolle-Dossal and Attouch-Cabot used in the context of fast gradient methods. We prove that the algorithm exhibits convergence rates of $O(1/k^2)$ for the primal-dual gap, the feasibility measure and the objective function value along the generated iterates, and also weak convergence of the primal-dual iterates to a primal-dual optimal solution.

The case of a nonsmooth objective function is treated as a special case of the problem of finding the zeros of the sum of a maximally monotone operator and a monotone and Lipschitz continuous operator. To solve the latter, we propose an algorithm with momentum and correction terms, which is shown to exhibit convergence rates of $o(1/k)$ for both the discrete velocity and the tangent residual expressed in terms of the last iterate, as well as weak convergence of the iterates to a solution of the monotone inclusion. When applied to the constrained optimization problem, the resulting algorithm exhibits convergence rates of $o(1/k)$ for the primal-dual gap, the feasibility measure and the objective function value along the generated iterates, and also weak convergence of the primal-dual iterates to a primal-dual optimal solution.

# The Limit Dynamic of the Su-Boyd-Candès Inertial Gradient System: A High-Resolution Analysis of Accelerated Optimization

Samir Adly

Université de Limoges

**Abstract.** This talk investigates the Su-Boyd-Candès (SBC) inertial gradient dynamics [1], known as $(AVD)_\alpha$, in the context of convex optimization and accelerated first-order methods. The $(AVD)_\alpha$ model provides a low-resolution ODE interpretation of Nesterov's accelerated gradient (NAG) method, involving an asymptotic vanishing damping term of the form $\alpha/t$ with $\alpha \geq 3$. By analyzing the convergence behavior of $(AVD)_\alpha$ when $\alpha \to +\infty$, we reveal its limit dynamics as a transformation to a first-order steepest descent, exhibiting singular perturbation behavior as it transitions from second to first-order dynamics. This tuning of $\alpha$ is critical, yielding a convergence rate of $o(1/t^2)$ for general convex functions and $O(1/t^{2\alpha/3})$ for strongly convex functions, achieving faster convergence as $\alpha$ grows. Our high-resolution ODE approach compares $(AVD)_\alpha$ with both Nesterov and Ravine methods, elucidating the conditions that yield optimal convergence rates and constants $C_\alpha$ over finite time horizons. Numerical results support our theoretical findings, emphasizing the crucial role of the Hessian-driven damping and the time-scaling techniques in accelerated optimization. This talk is based on the papers [2, 3].

**References.**

[1] W. J. Su, S. Boyd, E. J. Candès, A differential equation for modelling Nesterov' s accelerated gradient method: theory and insights. Journal of Machine Learning Research 17 (2016) 1-43.

[2] S. Adly and H. Attouch. Accelerated Optimization through Time-scale Analysis of Inertial Dynamics with Asymptotic Vanishing and Hessiandriven Dampings, Accepted for publication in Optimization Journal (2024).

[3] S. Adly and H. Attouch. Complexity analysis based on tuning the viscosity parameter of the Su-Boyd-Cand`es inertial gradient dynamics, Accepted for publication in Set-valued and Variational Analysis (2024).

# A Bundle-like View of Splitting Methods

Claudia Sagastizábal

IMECC-University of Campinas

**Abstract.** For large-scale optimization, popular approaches such as the ADMM and the Progressive Hedging algorithm exploit separable structures by solving in parallel individual sub-problems which are then coordinated by performing a simple algebraic step (typically, a projection onto a linear subspace).

While parallelism is the strength of all Douglas-Rachford-like splitting methods, their weak points are the adjustment of certain proximal parameter and the lack of a fully implementable stopping test. These difficulties, which date back to Spingarn's method of partial inverses, stem from the very design of these approaches, which were created to find zeroes of operators.

We discuss how to endow some splitting methods with an optimization perspective that introduces knowledge about the objective function throughout the iterative process. The resulting new family of methods à la bundle incorporates a projective step to coordinate parallel information while allowing the variation of the proximal parameters and the construction of a reliable stopping test. A Bundle Progressive Hedging algorithm, derived from the general theory, illustrates the interest of proposal.

Credit to co-authors will be given during the talk.

# A Unifying Framework for Markov Decision Process Models with a Bayesian Composite Risk Approach

Zhiping Chen

Xi’an Jiaotong University

**Abstract.** This paper develops a novel approach to modeling both epistemic and aleatoric uncertainties within Markov Decision Process (MDP) frameworks by utilizing a combination of two risk measures. The resulting model, named the Bayesian composite risk MDP, not only encompasses a variety of existing MDP configurations but also paves the way for the creation of innovative MDP models. We theoretically demonstrate, through convergence properties and quantitative estimates, that by integrating risk-averse strategies with dynamic Bayesian learning, our adaptive framework significantly improves decision-making under uncertainty. Specifically, it offers a more flexible and less conservative alternative compared to classical MDP models. Numerical experiments conducted on a finite horizon coin-flipping gambling problem and an infinite horizon inventory control problem demonstrate the effectiveness of our models for different cases.

# Heaviside Composite Optimization: A new paradigm in optimization

Jong-Shi Pang

University of Southern California

**Abstract.** This talk introduces the topic of Heaviside composite optimization and briefly covers its many facets: breadth in modeling, roles in old and new applications, theory of optimizers and stationary solutions, bridge with discrete optimization, and the progressive integer programming method. This research has benefitted from previous collaboration with Ying Cui (UC Berkeley), Yue Fang (CUHK-SZ), Shaoning Han (NUS), Junyi Liu (Tsinghua), and Xinyao Zhang (USC), and is presently being organized in a monograph co-authored with Junyi Liu.

# A Low-rank ALM for Doubly Nonnegative Relaxations of Mixed-binary QP

Kim-Chuan Toh

National University of Singapore

**Abstract.** Doubly nonnegative (DNN) programming problems are known to be challenging to solve because of their huge number of $\Omega(n^2)$ constraints and $\Omega(n^2)$ variables. In this work, we introduce RNNAL, a method for solving DNN relaxations of large-scale mixed-binary quadratic programs by leveraging their solutions' possible low-rank property. RNNAL is a globally convergent Riemannian based augmented Lagrangian method (ALM) that penalizes the nonnegative and complementarity constraints while preserving all other constraints as an algebraic variety. After applying the low-rank decomposition to the ALM subproblem, its feasible region becomes an algebraic variety with favorable geometric properties. Our low-rank decomposition model is different from the standard Burer-Monteiro (BM) decomposition model in that we make the crucial step to equivalently reformulate most of the quadratic constraints after the BM decomposition into fewer and more manageable affine constraints. This modification is also important in helping us to alleviate the violation of Slater's condition for the primal DNN problem. Moreover, we make the crucial step to show that the metric projection onto the algebraic variety, although non-convex, can be transformed into a solvable convex optimization problem under certain regularity conditions. Numerous numerical experiments are conducted to validate the efficiency of the proposed RNNAL method. [Joint work with Di Hou and Tianyun Tang]

# Proximal Gradient Methods in Nonsmooth Nonconvex Minimax: A Unified Convergence Analysis Framework

Marc Teboulle

Tel Aviv University

**Abstract.** We focus on nonsmooth nonconvex/strongly concave minimax problems which are prevalent in modern applications, thus departing from the more common weakly convex/concave and smooth models assumed in the recent literature. We present proximal gradient methods (parallel and alternating schemes) and show that both methods can be analyzed through a single scheme within a unified framework. In contrast to the current research which focuses on the complexity of obtaining only "nearly approximate" stationary solutions, here we derive pointwise global convergence to critical points, and as by-product, we prove refined complexity results. Furthermore, our approach being very flexible allows to expand the scope of minimax problems that can be addressed through the use of Non-Euclidean proximal steps, and to extend the convergence and complexity results to this broader setting. This talk is based on joint work with Eyal Cohen.

# On the Theory of Maximal Monotone Operators in General Banach Spaces

Andrew Eberhard

Royal Melbourne Institute of Technology

**Abstract.** We begin by discussing "closure operations" based on the convergence of bounded nets in on the product of the X, a real Banach space, endowed with the strong topology and its dual endowed with the weak star topology, for convex subsets of this product space. We discuss the relevance to the study of monotone operators defined on possibly non-reflexive, real Banach spaces.

We then develop a new epi-convergence based on this "convergence" of bounded nets and study the propagation of the associated variational convergences through conjugation of convex functions defined on this product space. These results are then applied to the problem of construction of a bigger-conjugate representative function for the recession operator associated with a maximal monotone operator on real Banach space. This is then used to study the relationship between the recession operator of a maximal monotone operator and the normal-cone operator associated with the closed, convex hull of the domain of that monotone operator. This allows us to show that the strong closure of the domain of any maximal monotone operator is convex in a general real Banach space.

Joint work with R. Wenczel.

# Descent Sequences in Weakly Convex Optimization

Mikhail Solodov

Institute for Pure and Applied Mathematics (IMPA)

**Abstract.** We present a framework for analyzing convergence and local rates of convergence of a class of descent algorithms, assuming the objective function is weakly convex. The framework is general, in the sense that it combines the possibility of explicit iterations (based on the gradient or a subgradient at the current iterate), implicit iterations (using a subgradient at the next iteration, like in the proximal schemes), as well as iterations when the associated subgradient is specially constructed and does not correspond neither to the current nor the next point (this is the case of descent steps in bundle methods). Under the subdifferential-based error bound on the distance to critical points, linear rates of convergence are established. Our analysis applies, among other techniques, to prox-descent for decomposable functions, the proximal-gradient method for a sum of functions, redistributed bundle methods, and a class of algorithms that can be cast in the feasible descent framework for constrained optimization.

# A New Dual Semismooth Newton Method for Polyhedral Projections

Chao Ding

Chinese Academy of Sciences

**Abstract.** We propose a dual semismooth Newton method for computing the orthogonal projection onto a given polyhedron, a fundamental optimization problem that serves as a critical building block for numerous important applications, e.g., financial risk management, statistics, and machine learning. Classical semismooth Newton methods typically depend on subgradient regularity assumptions for achieving local superlinear or quadratic convergence. Our approach, however, marks a significant breakthrough by demonstrating that it is always possible to identify a point where the existence of a nonsingular generalized Jacobian is guaranteed, regardless of any regularity conditions. Furthermore, we explain this phenomenon and its relationship with the weak strict Robinson constraint qualification (W-SRCQ) from the perspective of variational analysis. Building on this theoretical advancement, we develop an inexact semismooth Newton method with superlinear convergence for solving the polyhedral projection problem.

# New IPM for Linear Optimization Based on Affine-Scaling Directions in Parabolic Target Space

Yurii Nesterov

Corvinus University and partly SDS CUHK (SZ)

Professor emeritus at UCLouvain

**Abstract.** In this talk, we suggest new Interior-Point Method for Linear Optimization, based on idea of Parabolic Target Space. Our method can start at any strictly feasible primal-dual pair and go directly towards the solution by a long-step predictor-corrector scheme. Each iteration needs inversion of a matrix in small dimension. The worst-case complexity bound for the number of matrix factorizations is proportional to the square root of the dimension.

In our preliminary computational experiments, the actual number of iterations depends logarithmically on the dimensions. Method demonstrates a local superlinear convergence. In the end, the size of predictor step typically achieves 99.97% of the maximal possible step to the boundary. Contrary to the standard framework, our method is based on hyperbolic self-concordant barrier functions.

Joint work with M. Nagy, T. Illes, and P. Rigo (Corvinus).

# Control Systems on Wasserstein Spaces

Hélène Frankowska

CNRS, Institut de Mathématiques de Jussieu - Paris Rive Gauche, and Sorbonne Université

**Abstract.** Dynamical systems involving large number of agents can be approached by considering moving sets of agents instead of union of vector-valued timedependent paths of individual agents.

In some social sciences models, like the evacuation one, it may be interesting to assign to sets their measures and to work with metric spaces of Borel probability measures, the so called Wasserstein spaces. Here translation of vectors is replaced by the push-forward operation on measures leading to natural extensions of notions of nonsmooth analysis to this metric space.

The dynamics of measures can be described then via controlled continuity (transport) equations. For Lipschitz kind dynamics, some cornerstone results of classical control theory known in the Euclidean framework have their analogues in Wasserstein spaces.

In this talk I will present necessary and sufficient conditions from [1] for the existence of solutions to state-constrained control systems. Then I will recall more elaborate results from [2] for measures with compact support that were applied to investigate uniqueness of solutions to Hamilton-Jacobi-Bellman equations.

[1] Bonnet B., Frankowska H.,

On the Viability and Invariance of Proper Sets under Continuity Inclusions in Wasserstein Spaces, SIAM Journal on Mathematical Analysis, to appear.

[2] Badreddine Z. , Frankowska H.,

Solutions to Hamilton-Jacobi equation on a Wasserstein space, Calculus of Variations and PDEs, 81: 9, 2022.

# The Elicited Progressive Decoupling Algorithm: on the Rate of Convergence and the Choice of Parameters

Jie Sun

Curtin University

**Abstract.** We study the progressive decoupling algorithm (PDA) of Rockafellar and focus on the elicited version of the algorithm. Based on a generalized Yosida-regularization of Spingarn's partial inverse of an elicitable operator, it is shown that the elicited progressive decoupling algorithm (EPDA), in a particular nonmonotone setting, linearly converges at a rate that could be viewed as the rate of a rescaled PDA, which may provide certain guidance to the selection of the parameters in computational practice. A preliminary numerical experiment shows that the choice of elicitation constant indeed makes an impact on the EPDA's efficiency.

This is a joint work with Professor Min Zhang of Guangzhou University, PRC.

# Approximations of Rockafellians, Lagrangians, and Dual Functions

Johannes O. Royset

University of Southern California

**Abstract.** Solutions of an optimization problem are sensitive to changes caused by approximations or parametric perturbations, especially in the nonconvex setting. This presentation investigates the ability of substitute problems, constructed from Rockafellian functions, to provide robustness against such approximations. Unlike classical stability analysis focused on local changes around (local) minimizers, we employ epi-convergence to examine whether the approximating problems suitably approach the actual one globally. We show that under natural assumptions the substitute problems can be well behaved in the sense of epi-convergence even though the actual one is not. We further quantify the rates of convergence that often lead to Lipschitz-kind stability properties for the substitute problems.

# A Neural Network Approach to Learning Solutions of a Class of Elliptic Variational Inequalities

Michael Hintermüller

Humboldt-Universität zu Berlin

**Abstract.** We develop a weak adversarial approach to solving obstacle problems using neural networks. By using (generalized) regularized gap functions and their properties we rewrite the obstacle problem (which is an elliptic variational inequality) as a minmax problem, providing a natural formulation amenable to a learning approach. Our approach, in contrast to much of the literature, does not require the elliptic operator to be symmetric. We provide an error analysis for suitable discretisations of the continuous problem, estimating in particular the approximation and statistical errors. Parametrising the solution and test function as neural networks, we apply a modified gradient descent ascent algorithm to treat the problem and conclude the paper with various examples and experiments. Our solution algorithm is in particular able to easily handle obstacle problems that feature biactivity or lack of strict complementarity, a situation that poses difficulty for traditional numerical methods.

# Optimization with Superquantile Constraints: A Fast Computational Approach

Ying Cui

University of California, Berkeley

**Abstract.** We present an efficient and scalable second-order computational framework for solving large-scale optimization problems with superquantile constraints. Unlike empirical risk models, superquantile models have nonseparable constraints that make typical first-order algorithms difficult to scale. We address the challenge by adopting a hybrid of the second-order semismooth Newton method and the augmented Lagrangian method, which takes advantage of the structured sparsity brought by the risk sensitive measures. The key to make the proposed computational framework scalable in terms of the number of training data is that the matrix-vector multiplication in solving the resulting Newton system can be computed in a reduced space due to the aforementioned sparsity. The computational cost per iteration for the Newton method is similar or even smaller than that of the first-order method. Our developed solver is expected to help improve the reliability and accuracy of statistical estimation and prediction, as well as control the risk of adverse events for safety-critical applications.